

Shear localization and shear instability in materials in the ductile field

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Abstract—In the ductile field, instability of flow (in time) is necessarily associated with localization (in space), leading to shear zones at various scales, from slip lines and slip bands due to the glide of dislocations in glide planes of single crystals, to shear bands in polycrystals deformed in plane strain. The mechanics and physics of unstable, localized flow in simple shear are reviewed with special reference to experimental results on materials. Instability conditions taking into account strain hardening or softening, strain-rate sensitivity and shear heating are presented. Recent models for the growth of instabilities are reviewed. Physical mechanisms for flow softening are examined; geometrical softening, phase rearrangement in two-phases materials, phase transformation, adiabatic heating, and dynamic recrystallization. The relevance of these mechanisms to geological conditions is assessed.

INTRODUCTION

IN STRUCTURAL geology as well as in processing metallurgy, non-uniform deformation is the rule rather than the exception. There exists a hierarchy of scales in deformation whereby non-uniform (heterogeneous) deformation at one scale will be smoothed out and appear uniform at a larger scale; but there will be non-uniformities at that scale which in turn will be blurred out at a still larger scale, and so on. At the smallest scale, in a single crystalline grain, we have the quantum of deformation produced by one dislocation sweeping a slip plane; a train of dislocations on neighboring slip planes gives rise to a microscopically visible slip band or glide lamella. Strain localization in macroscopic deformation bands occurs when it becomes easier to further deform an already deformed zone than to initiate deformation elsewhere (by easier deformation it is meant that, for a given imposed strain rate, the applied stress will increase by a lesser amount). Strain localization is therefore the manifestation of instability in plastic deformation. It can occur in non-crystalline materials (polymers) as well as in polycrystalline materials such as metals or rocks. Whether it occurs depends on the mechanical conditions of straining, the material properties and the geometry of the deforming body; but not on its dimensions. In the field of tectonics we find deformation bands at the scale of a thin section, a hand specimen, an outcrop, a province or a continent. We may however assume that the underlying mechanics are the same as in the industrial processing of materials. Strain localization is a serious inconvenience in metallurgical forming operations (e.g. rolling or deep drawing) since it can lead to failure and waste of material. It is therefore natural that most investigations on the occurrence of strain localisation or, equivalently, plastic instability have been published in the metallurgical literature. Since there have been recent reviews of the mechanical models for the inception and growth of plastic instability (Backofen 1972, Argon 1973, Poirier 1976) as well as of the mate-

rial properties leading to it in the case of metals (Jonas & Luton 1978), the present paper is not to be considered as an exhaustive review.

In what follows I will set the problem of determining the conditions for shear instability, and hence shear zone formation, in the more general framework of plastic instability under tensile conditions and I will write a general condition for instability, every term of which has a physical meaning. In a second part the relevance of all the terms to the geological problem of shear zone formation will be discussed in relation to metallurgical examples. Finally, two models of growth of shear instability will be discussed and applied to geological problems.

A CRITERION FOR INSTABILITY

Although various criteria have been proposed in the literature (see Poirier 1976, Jonas *et al.* 1976) we will limit ourselves here to the least sophisticated one which readily allows the discussion of all the instability producing factors.

Let us consider a sample of a given material undergoing a uniaxial tensile test. We will assume that the sample has the shape of a rod or a slab with a minimal cross sectional area A (there can be small area fluctuations); ϵ and σ are the engineering axial strain and stress and $\dot{\epsilon} = \frac{d\epsilon}{dt}$ the imposed constant strain rate. The applied load at a given time, $F = \sigma A$, obviously represents the load-carrying capacity of the sample. The material deforms plastically by shear according to the constitutive equation (for 1 slip system of a single crystal):

$$\tau = \tau_0 \gamma_i^n \dot{\gamma}_i^m \exp \frac{mQ}{RT} \quad (1)$$

where τ is the resolved shear stress and γ_i and $\dot{\gamma}_i$ respectively the resolved shear strain and shear strain-rate, on the slip plane, in the slip direction. τ_0 is a constant characteristic of the strength on the slip system; it has the

dimensions of (stress) \times (time)^{*m*}. *Q* is the activation energy of steady-state creep at constant stress (no strain hardening, hence *n* = 0) whose constitutive equation is usually given as:

$$\dot{\gamma} = \dot{\gamma}_0 \tau \exp - \frac{Q}{RT} \quad (2)$$

In the case of a polycrystal, we can introduce the Taylor factor *M* (Kocks 1958) which depends on the orientations of the active slip systems in the grains with respect to the tensile axis. If we assume that all slip systems are equivalent and active under the same resolved shear stress τ , the Taylor factor is defined by:

$$\sigma = M\tau$$

or

$$\sum_i \gamma_i = M\epsilon.$$

We can write (Dillamore *et al.* 1979):

$$\tau = \tau_0 (\sum_i \gamma_i)^n (\sum_i \dot{\gamma}_i)^m \exp \frac{mQ}{RT} \quad (3)$$

or

$$\sigma = KT_0 \epsilon^n \dot{\epsilon}^m \exp \frac{mQ}{RT} \quad (4)$$

where $K = M^{(1+n+m)}$.

Thus $n \equiv \frac{\partial \ln \sigma}{\partial \ln \epsilon}$ is the strain hardening coefficient and $m \equiv \frac{\partial \ln \sigma}{\partial \ln \dot{\epsilon}}$ is the strain rate sensitivity of the stress.

I choose for the instability criterion the admittedly crude condition that the load carrying capacity *F* decrease with increasing strain. This will lead to localization of strain which will tend to concentrate where further straining can be done at a lower stress. The condition is conveniently expressed as:

$$\frac{d \ln F}{d \epsilon} \leq 0 \quad (5)$$

or, since $d \ln F + d \ln \sigma + d \ln A$, as:

$$\frac{d \ln \sigma}{d \epsilon} \leq - \frac{d \ln A}{d \epsilon} \quad (6)$$

Using equations (3) and (4), we obtain:

$$(1+n+m) \frac{d \ln M}{d \epsilon} + \frac{d \ln \tau_0}{d \epsilon} + \frac{n}{\epsilon} - m \frac{d \ln \dot{\epsilon}}{d \epsilon} - \frac{mQ}{RT^2} \frac{dT}{d \epsilon} \leq - \frac{d \ln A}{d \epsilon} \quad (7)$$

The term $-\frac{d \ln A}{d \epsilon}$ on the right-hand side (RHS) of (6) and (7) is always positive in tension. For rod-shaped samples it is equal to 1 and equation (6) is then written as:

$$\frac{d \sigma}{d \epsilon} = \sigma.$$

This is the well-known Considère criterion by which, using the stress-strain curve, one can determine the maximum load-carrying capacity of a sample. Above the corresponding strain, necking will develop and lead to ductile failure. For slabs in tension (corresponding to the case of boudinage) one finds $-\frac{d \ln A}{d \epsilon} = \frac{1}{2}$ (Backofen 1972) when the neck thins down in the direction normal

to the slab. In the case of pure or simple shear, the cross sectional area remains constant and $\frac{d \ln A}{d \epsilon} = 0$. The

plastic flow can concentrate along the characteristics of the slip line field as soon as the material approximates the ideal plastic state (Chakrabarti & Spretnak 1975); the plane strain regime implied here is the only one which allows localized shear along unchanging surfaces (Backofen 1972). All terms on the left-hand side (LHS) of (7) correspond to strain-induced hardening or softening according to their sign. As stated by Backofen (1972): "The condition for initiating unstable flow after any amount of prior straining is that somewhere in the material, the next increment of strain-induced hardening be cancelled out by an accompanying strain-induced softening, then further straining will tend to concentrate in the plane where resistance to continuous flow is first lost." Instability can therefore occur in shear when the net balance of the terms on the LHS of (7) is negative. These terms respectively correspond to various physical processes, potentially leading to a maximum in the load-strain curve. From left to right they are: geometric softening, structural softening, strain softening, strain-rate softening, and thermal softening. We will investigate each of these in turn in the following section.

FLOW-SOFTENING MECHANISMS

Geometric softening

This occurs when during deformation the active slip planes rotate into more favourable orientations, that is, orientations for which the resolved shear stress is higher for a constant applied stress. The clearest examples are found in single crystals of hexagonal metals such as cadmium or zinc which glide easily on the basal plane. Boas & Schmid (1929) showed that the load could decrease by up to 60% in the case of single crystals of cadmium deformed in tension with the basal plane at an initial angle of 79° to the tensile axis. This behaviour is obviously linked to a decrease of *M* as ϵ increases ($\frac{dM}{d\epsilon} < 0$).

As discussed by Jonas & Luton (1978), such a dramatic decrease in load carrying ability with increasing strain is usually not observed in polycrystals, mainly because most processing operations lead to preferred orientations for which the Taylor factor *M* is minimal, hence to stable orientations which are not susceptible to soften more by further flow. However, geometric softening may occur to some extent in the case of some polycrystalline metals which have been hot worked in a high-temperature phase, and which after a solid phase transformation have inherited a preferred orientation unstable with respect to lower temperature flow (Jonas & Luton 1978). Dillamore *et al.* (1979) have attributed to geometric softening the formation of shear bands in heavily rolled cubic metals: they have computed the Taylor factor *M* as a function of θ , the angle between the shear bands and the maximum shear stress plane, for

every family of crystal orientations. The geometric softening term in (7) could thus be written $\frac{d \ln M}{d \epsilon} + \frac{d \ln M}{d \theta} \cdot \frac{d \theta}{d \epsilon}$, where $d \theta$ is the rate of crystal rotation with strain. They find that the actual shear-band angle corresponds to maximization of work softening through crystal rotation. There seems to be no reason why a similar process could not operate to some extent in rocks, given appropriate conditions. More generally some geometric softening may be present whenever a preferred orientation, unstable with respect to the actual flow conditions, has been inherited from a previous tectonic episode involving different slip systems (e.g. in quartz, basal or prismatic slip are respectively favored at lower or higher temperatures, leading to different preferred orientation). Rotation softening (Cobbold 1977) occurring in anisotropic rocks where the principal directions of anisotropy rotate during deformation may also be considered as geometric softening.

Structural softening

The effect of the structural parameters on σ is represented by the term τ_0 in (3). Structural softening occurs wherever the structure changes with strain towards a more easily deformable state ($\frac{d \ln \tau_0}{d \epsilon} < 0$). Two structural softening mechanisms, well documented in metals, may be relevant to the case of rocks: these are dynamic recrystallization and second-phase rearrangement. It is well known, since the work of Rossard & Blain (1958) on steel, that the presence of a maximum (or oscillations) in the load-strain curve can be attributed to the formation of new dynamically recrystallized grains. The softening depends on the kinetics of the recrystallization process and on the strain rate (Sellars 1978). Burrows *et al.* (in press) have recently investigated the dynamic recrystallization of a magnesium alloy (Magnox Al 80) which in many respects can be considered as a model system for quartz. For a strain-rate $\dot{\epsilon} = 10^{-5} \text{ s}^{-1}$ and temperatures between 260° and 370°C they found dynamic recrystallization associated with a peak in the stress-strain curve. Under the conditions leading to the steeper peak (260°C) they found that recrystallized bands merged into a single shear band which accommodated all subsequent deformation. The structural softening due to dynamic recrystallization was therefore the source of shear bands.

Dynamic recrystallization can however take place by two different mechanisms according to the range of temperature and applied stress: at low T and σ , the new grains are formed by rotation of the polygonized subgrains; whereas at high T and σ , migration of the grain boundaries takes place in addition (Guillopé & Poirier 1979, Poirier & Guillopé 1979). Only the latter mechanism involves flow softening and can lead to shear zone formation.

Another source of structural softening exists in two phase materials, the evolution with strain of the phase structure towards a weaker one (Jonas & Luton 1978).

In some cases, like the globularization of lamellar

structures or the coarsening of fine precipitates, the evolution towards a more globular or coarser structure results from the associated decrease in surface energy and would therefore be possible even without strain as long as the material is held at a sufficient temperature for a long enough time. However, the evolution is usually considerably enhanced by the presence of moving dislocations associated with straining. In other cases, the structural evolution is directly caused by strain, as when a hard continuous connected phase is broken down into equiaxed particles. The break up of a β -phase network in an alloy of zirconium. 2.5% Niobium thus produces considerable softening (Jonas & Luton 1978). Shear zones along the maximum resolved shear-stress plane have been observed in a fine-grained equivolumic synthetic alloy of iron and silver, uniaxially compressed at room temperature (Le Hazif pers. comm.) (Fig. 1). The phases were originally distributed as two intermeshed networks of Fe and Ag; in the shear zones the networks were broken and the phases tended to assume the layered configuration found in samples uniformly deformed to large strains (Le Hazif 1978).

Strain softening/hardening

We are dealing here with the third term of equation (7), $\frac{n d \ln \dot{\epsilon}}{d \epsilon} = \frac{n}{\dot{\epsilon}}$, which represents strain softening *sensu stricto* if $n < 0$. Geometric and structural softening, although brought about by strain did not result directly from the operation of the strain mechanism. In most cases n is positive, and this term corresponding to work hardening has to be balanced by other negative terms in the LHS of (7) for shear instability to be possible. However, for high temperatures and large strains n usually tends to vanishingly small values and the material behaves as an ideally plastic solid, which if it does not soften at least does not harden. Nevertheless, there can be real work softening ($n < 0$) when a yield point is present on the stress-strain curve at the beginning of deformation. This is the case in many minerals as well as metals and covalent solids (e.g. quartz, lithium fluoride, ferrite, silicon) when there are not enough mobile dislocations at the beginning of deformation, either because they are pinned by impurity atoms (as in ferritic steel or more generally body-centred cubic metals), or because they must overcome a high Peierls energy barrier to glide over to the next valley. In both cases the stress goes up until it reaches the Peierls stress or the unpinning stress when dislocations are freed in avalanches, thus producing unstable strain under decreasing stress in slip bands. We will see later in the case of quartz how the negative slope of the stress-strain curve defines an instability region where shear bands can grow.

Strain-rate softening/hardening

In most cases the term $m \frac{d \ln \dot{\epsilon}}{d \epsilon}$ is positive and here too we deal with a hardening term which must be balanced

by other, negative, terms for instability to be possible. As we are concerned here with deformation at constant strain rate the important parameter is the strain-rate sensitivity of the stress, m . For a Newtonian viscous material, $m = 1$, and this term is not destabilizing. Many materials behave as non-Newtonian viscous fluids with $m \approx 0.2$ to 0.3 ; they are inherently less stable and more prone to shear localization, but as long as m is not negative, the term $m \frac{d \ln \dot{\epsilon}}{d \epsilon}$ cannot by itself promote instability.

Chung *et al.* (1977) have recently reported unstable shear failure of a quenched Al-Zn-Mg alloy in uniaxial tension at room temperature; the envelope of the stress-strain curve of various strain rates decreased with increasing strain rate, thus defining a region of negative strain-rate sensitivity where shear failure occurred.

Thermal softening

The last term of the LHS of equation (5) is always a softening term; at worst it is equal to zero if the deformation is athermal, or when the deformation is thermally activated if the temperature does not increase significantly during straining. The value of the term $\frac{dT}{d\epsilon}$ can easily be estimated (Argon 1973). If $\alpha < 1$ is the percentage of strain energy transformed into heat, ρ and C_p respectively being the density and heat capacity at constant pressure and if there is no heat exchange (adiabatic case), we have:

$$\alpha \sigma d\epsilon = \rho C_p dT.$$

Hence in the general case:

$$\frac{dT}{d\epsilon} \leq \frac{\alpha \sigma}{\rho C_p}.$$

If we consider the heat as being produced in a conducting slab of thickness h , we have for the maximum temperature:

$$\rho C_p \frac{dT}{d\epsilon} \approx \alpha \sigma \frac{h^2 \dot{\epsilon}}{\kappa \epsilon}$$

where κ is the thermal diffusivity.

The term $\frac{dT}{d\epsilon}$ will be important only if the material has a low thermal conductivity and if the strain rate is quite high. For metals, which have a high conductivity, thermal softening may be neglected only for $\dot{\epsilon} > 1 \text{ s}^{-1}$ or at very low temperatures when the heat capacity becomes very small. An interesting case of shear instability caused by shear heating has been reported in pure niobium single crystals by Kubin & Jouffrey (1971) at liquid helium temperatures (Fig. 2). Rocks do not evacuate heat as rapidly as metals and we will examine later the importance of thermal softening in the framework of a particular model for shear-zone growth. If straining occurs at temperatures not far below the temperature for a phase transformation or a reaction of dehydration in minerals, it is conceivable that even a slight temperature increase might be enough to drive the

system through the transition and possibly induce a catastrophic softening by transformation plasticity or fluid weakening (White & Knipe 1978), hence making shear instabilities possible.

GROWTH KINETICS OF SHEAR BANDS AND ZONES

We have written a criterion for plastic instability (7) in shear which expresses the conditions that the material must fulfill for shear localization to be possible. If we consider the shear strain γ rather than the engineering strain ϵ , as we must if we focus on the formation of a shear band, the RHS of (7) is equal to zero and we can write (in the isotropic case: $M = 1$).

$$-\frac{d \ln \tau_0}{d \gamma} + \frac{n}{\gamma} + m \frac{d \ln \dot{\gamma}}{d \gamma} - \frac{m Q}{RT^2} \frac{dT}{d \gamma} \leq 0. \quad (8)$$

This condition expresses that if for some reason (like a non-uniformity in strength τ_0 , or in temperature) there is locally a slight fluctuation in shear strain, it will be amplified as strain increases. However, this does not mean that we will obtain a characterized shear band after any amount of strain; a shear band is recognized as such if the shear strain in the band is noticeably larger than the uniform strain outside. The shear band must therefore be fully developed before the uniform shear strain becomes so large that a local variation is hardly noticeable or before the material conditions leading to instability disappear as strain increases (structural softening or strain softening are almost always transient). We therefore need a kinetic model for the growth of the instability. We will briefly review two models: one by Bowden (1970) was developed to account for inhomogeneous deformation of polymers but can be applied to any strain-softening material; the other by Poirier *et al.* (1979) applies to the case of simple shear in rocks a metallurgical model (Jonas *et al.* 1976) for plastic instability. Both follow the growth of shear bands or zones from initial heterogeneities in a medium homogeneously strained at constant imposed shear-strain rate.

A model for inhomogeneous deformation in polymers (Bowden 1970)

The material considered has a stress strain curve which shows a region where the stress drops as the strain increases ($n < 0$), that is where strain softening occurs. This is the only source of softening; there is no structural softening, a slight strain rate hardening ($m > 0$ but small) and the deformation is athermal, hence no thermal softening. The material is deforming homogeneously in shear with an imposed shear rate $\dot{\gamma}_0$ but contains embryonic bands where the local shear strain is larger by $\Delta\gamma$. Assuming that the stress remains uniform during the initial growth, it is possible to follow the development of the divergence in strain and strain rate between the bands and the matrix of the surface $\sigma = f(\gamma, \dot{\gamma})$

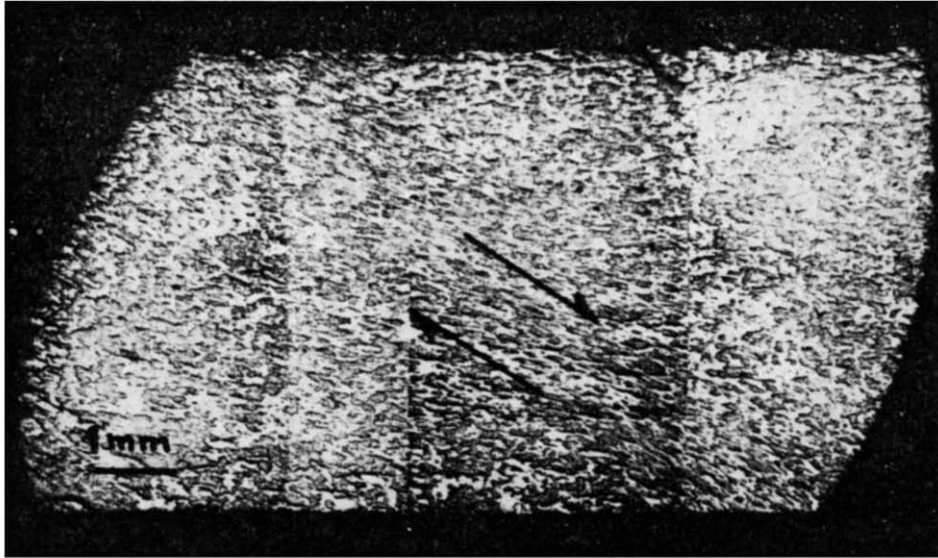


Fig. 1. Shear zone on the maximum resolved shear stress plane in uniaxial compression ($\epsilon = 10^{-1} \text{ s}^{-1}$; $T = 25^\circ\text{C}$) of a phased specimen prepared by hot isostatic pressing of 30 micron powders of iron and silver (light- silver; dark- iron). Notice the layering in the shear zone (Courtesy R. Le Hazif).

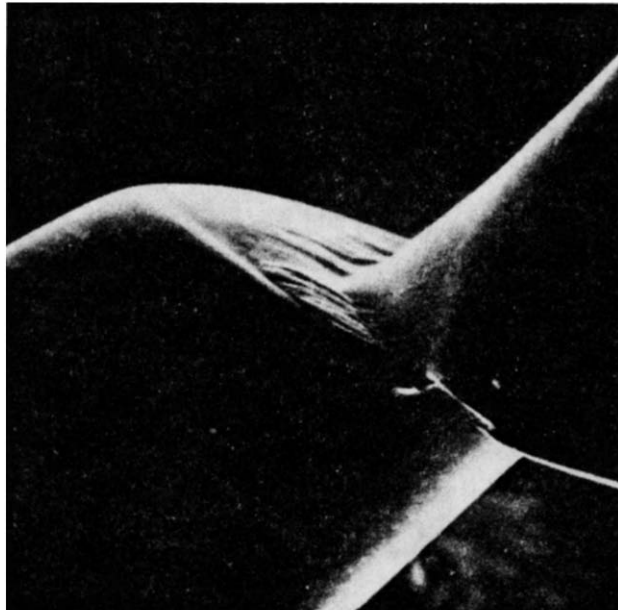


Fig. 2. Shear zone due to shear heating in a single crystal of niobium during uniaxial tension at $T = 4.2 \text{ K}$. Diameter of sample is 2 mm (courtesy L. P. Kubin).

representative of the material properties. One can calculate the overall strain that has to be applied to the specimen before the band strain-rate will double:

$$\gamma_m^D = \gamma^* \ln \left(\frac{\dot{\gamma}^*}{\Delta \dot{\gamma}} \right) \quad (9)$$

where $\gamma^* = \dot{\gamma}_0$ is a characteristic strain, and $G \equiv \left. \frac{\partial \dot{\gamma}}{\partial \sigma} \right|_{\sigma}$ is the slope of the constant-stress contours on the $\sigma = f(\gamma, \dot{\gamma})$ surface. The parameters that control the growth of the instability are therefore the characteristic strain γ^* and the initial strain inhomogeneity $\Delta \gamma$. If the magnitudes of γ^* and $\Delta \gamma$ can be known or estimated it is thus possible to predict whether shear instabilities are likely to develop in the material. As there is nothing in the model specific to polymers, it can be applied to the growth of deformation bands in a single crystal or of shear zones in a rock, provided the mineral or the rock exhibits a stress drop on its stress-strain curve, that is a region of negative slope on the $\sigma = f(\gamma, \dot{\gamma})$ surface. This is the case of quartz single crystals (Balderman 1974, Morrison-Smith *et al.* 1976) whose $\sigma = f(\gamma, \dot{\gamma})$ surface can be represented as in Fig. 3.

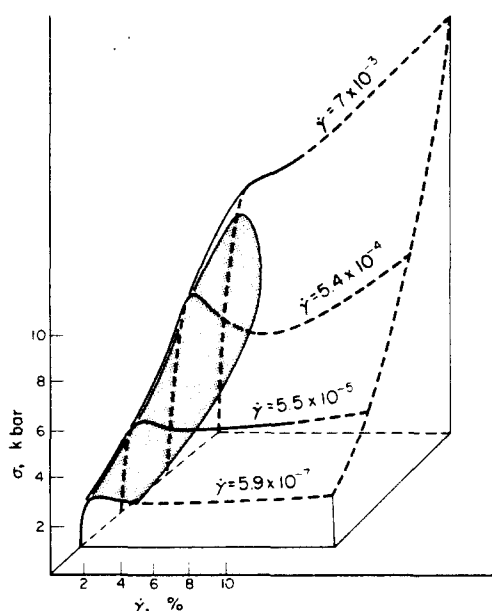


Fig. 3. Surface $\sigma = f(\gamma, \dot{\gamma})$ for single crystalline quartz. Stress σ in kbar (1 kbar = 10^8 Pa), strain γ in %, strain rate in s^{-1} . $T = 500^\circ C$ (from the results of Balderman 1974).

Balderman's results for experiments at $T = 500^\circ C$ and various strain rates can be used to estimate the characteristic strain γ^* for a typical geological strain-rate, $\dot{\gamma} = 10^{-14} s^{-1}$. We can write:

$$\gamma^* = \dot{\gamma}_0 \left[\left(\frac{\partial \dot{\gamma}}{\partial \sigma} \right) \left(\frac{\partial \sigma}{\partial \dot{\gamma}} \right) \right]^{-1} \quad (10)$$

The negative slope of the stress-strain curves can be measured and extrapolated to $\dot{\gamma}_0 = 10^{-14} s^{-1}$, thus giving the required estimate $\partial \sigma = 10^{-2}$ (Fig. 4). The value of the stress at the yield point is related to the strain rate by the relation given by Balderman:

$$\dot{\gamma} = 5.82 \times 10^{-7} \cdot \sigma_y^{3.64} \quad (\sigma_y \text{ in kbar; } 1 \text{ kbar} = 10^8 \text{ Pa}).$$

Reasonably assuming that the same relation holds for the stress in the unstable region, we have:

$$\frac{\partial \dot{\gamma}}{\partial \sigma} = 2.12 \times 10^{-6} (1.72 \times 10^6 \dot{\gamma}_0)^{0.725}.$$

For $\dot{\gamma}_0 = 10^{-14} s^{-1}$, this gives $\frac{\partial \dot{\gamma}}{\partial \sigma} = 4.99 \times 10^{-12}$, (s^{-1}

$kbar^{-1}$) and (10) gives $\gamma^* = 0.2$.

If there is a strain heterogeneity $\Delta \gamma = 0.01$ in an embryonic band, the band strain-rate could double for a overall strain of $\gamma_m^D = 0.6$, given by (9).

As can be seen in Fig. 3, the strain γ_m^D is larger than the strain for which the system has ceased to be unstable. For $\Delta \gamma \approx 0.17$, $\gamma_m^D = 0.04$ which is well in the unstable part of the curve. Morrison-Smith *et al.* (1976) have shown that deformation bands in quartz are correlated with a higher OH content; thus inhomogeneity in water content can cause local strain variation and shear bands in quartz grains, even in geological conditions. The initial inhomogeneity in shear strain can be replaced by an inhomogeneity in strength as we will see in the following model.

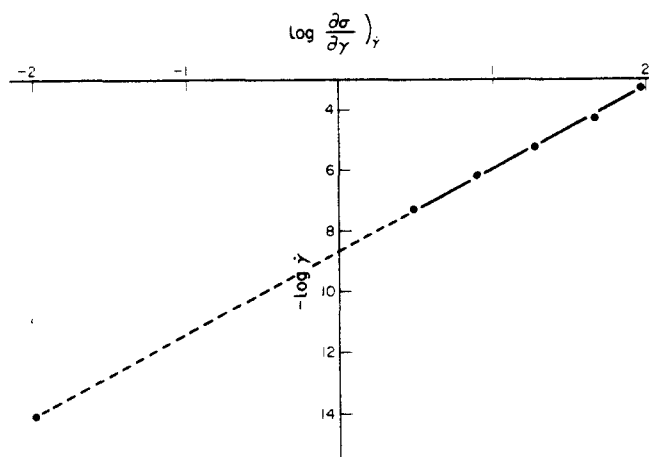


Fig. 4. $\partial \sigma$ as a function of $\dot{\gamma}$ for single crystalline quartz, from the results of Balderman (1974). The straight line is extrapolated to $\dot{\gamma} = 10^{-14} s^{-1}$.

A model for growth of shear zones in rocks (Poirier et al. 1979)

The starting point of this model is the previous model of Jonas *et al.* (1976) for plastic instability in tension or compression in metals, adapted to the case of shear (no area variation). The constitutive equation adopted for the material is that corresponding to an almost perfectly plastic rheology, that is after a very small strain, the applied shear stress at constant strain rate saturates to a constant value; this value, however, depends on the imposed strain rate and on temperature. The constitutive equation therefore has the same form as (3) with $K = 1$. Assuming a constant imposed shear-strain rate, the condition of instability is written by expressing that the variation of the shear stress σ along the direction y perpendicular to the shear zone is equal to zero at any given time.

$$\left(\frac{\partial \ln \sigma}{\partial y}\right)_t \equiv \frac{\partial \ln \sigma}{\partial y} \frac{\partial \gamma}{\dot{\gamma} \partial \gamma} + m \frac{\partial \ln \dot{\gamma}}{\partial y} + \frac{\partial \ln \tau_0}{\partial y} = 0. \quad (11)$$

Introducing the gradient of strain along λ , $\equiv \left(\frac{\partial \gamma}{\partial y}\right)_t$ and its

derivatives with respect to strain λ' and λ'' , (11) can be written:

$$m\lambda' = -\Gamma\lambda - \frac{\partial \ln \tau_0}{\partial y} \quad (12)$$

where $\Gamma \equiv \frac{\partial \ln \sigma}{\partial y}$ is the hardening coefficient.

The deformation is unstable if the strain gradient increases with strain, that is if $\lambda' > 0$. To ensure that the instability does not disappear at large strains, one needs the supplementary condition $\lambda'' \geq 0$. Starting from chosen initial conditions (T_0 , $\dot{\gamma}_0$) and assuming that the embryonic shear zone is characterized by a strength defect $\Delta\tau_0$, it is possible to calculate numerically the values of strain, stress and temperature in the shear zone for successive time increments. Shear heating is taken into account and causes the hardening coefficient Γ to become negative (thermal softening). The model would also allow structural softening to be taken into account by making the strength τ_0 decrease at each step. Application of this model to the case of granite for a geological strain rate of $\dot{\gamma}_0 = 10^{-14} \text{ s}^{-1}$ and two initial temperatures (327° and 527°C) shows the importance of the relative strength defect $\Delta\tau_0$ in giving a clear cut shear zone. If there is no structural softening, that is if the strength defect stays at its initial value, the strain in the shear zone will tend to increase only linearly with the homogeneous strain outside; consequently the difference in the shear angle $\varphi = \text{tg}^{-1}\gamma$ inside the zone and outside will tend to be rather small at large strains, so that the shear zone will be less clear-cut than is usually the case in the field. Some sort of structural softening is therefore probably important in the production of well-defined shear zones. On the contrary, the effect of thermal softening does not seem to be important when the uniform shear-strain rate is assumed to be constant. At the lower initial temperature (327°C), shear heating causes the temperature to increase and the thermally activated stress to decrease, which in turn causes shear heating to decrease. At the higher temperature (527°C) the effect of shear heating is anyway very small. Under these conditions, for a given shear angle outside the zone (at a given time), the shear angle inside the zone will be approximately the same for adiabatic deformation at

327°C and for adiabatic or isothermal deformation at 527°C. It is therefore unlikely that thermal runaway leading to melting can occur under conditions of constant geological strain rate and plastically relaxing stress.

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